Journal of Sound and Vibration (2000) **237**(2), 357–360
doi:10.1006/jsvi.1999.3005, available online at http://www.idealibrary.com on IDEAL®

LETTERS TO THE EDITOR

COMMENTS ON "IDENTIFICATION OF MULTI-DEGREE-OF-FREEDOM NON-LINEAR SYSTEMS UNDER RANDOM EXCITATIONS BY THE 'REVERSE PATH' SPECTRAL METHOD"

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(*Received* 19 *August* 1999)

The authors of reference [1] are to be commended for implementing the "reverse path" non-linear spectral analysis method for identifying the constituents elements of simulated three- and five-degree-of-freedom (d.o.f.) non-linear systems. However, we feel that the paper requires some comment as to origins of the analysis methods used and the originality of their method. In the first instance, the attribution of the "reverse path" modelling technique to Bendat [2] is misleading. The actual technique for remodelling single-d.o.f. systems in this way was first proposed by Rice and Fitzpatrick [3] and has since been applied to a number of systems using both simulated data [4, 5] and actual experimental data from a system with a linear element and four non-linear elements [5, 6]. This work was, in fact, referred to by Bendat in his book on non-linear system identification [2].

The second issue is that the authors refer incorrectly to a paper by Rice and Fitzpatrick [7] which extended the original non-linear spectral analysis (or "reverse path") method to multi-d.o.f. systems. The authors state: "A similar approach has been used for the identification of two-d.o.f. non-linear systems where each response location is considered as a single d.o.f. mechanical oscillator". This is incorrect as the complete coupled system was considered and the identification was for this. The approach of Richards and Singh is identical to our method using essentially, a force balance and the various block diagrams in their paper are similar to those already published. In addition, the equations given by us as (1), (2) and (3) are identical to those used by Richards and Singh. Our equations are completely general and make no assumption about the actual location of the non-linearities. The only requirement is that the analysis is for discrete multi-d.o.f. configurations. Indeed, whereas the method proposed by Rice and Fitzpatrick does not make any *a priori* assumptions about the locations of the non-linearities, that of Richards and Singh requires that the locations of the non-linearities are known in order to accommodate local linear sub-models. Identification of the location of non-linearities in real structures is, of course, a further issue which has been addressed, for example, by Xu and Rice [8].

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AUTHORS' REPLY

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(*Received* 28 *February* 2000)

For an historical evolution of the "reverse path" spectral method in the context of single-degree-of-freedom (d.o.f.) systems, the interested reader can and should refer to the extensive literature review provided by Bendat [1]. For instance, a quote from this citation [1] concerning the work of early investigations follows: "Rice and Fitzpatrick [1988, 1991] wrote two outstanding articles dealing with useful techniques for non-linear system analysis and identification that were developed following the works of Bendat and Piersol [1982, 1986a], Bendat [1983], and Vugts and Bouquet [1985], but independently of other related work by Bendat [1985], Bendat and Palo [1989, 1990], and Bendat *et al.* [1990, 1992]."

Since we developed our method for specific application to multi-d.o.f. systems [2], we did not feel the need to duplicate the extensive literature review on the identification of single-d.o.f. systems. Formulation of the "reverse path" spectral technique has been thoroughly covered by Bendat in a well-known textbook [3]; therefore, it is unnecessary to cite all parties involved with the development and application of the method. Consequently, only the most recent work on the development of the method has been referenced in our publications [2, 4] so that readers may adequately follow the analytical treatment. In summary, we have followed the accepted practices of citing and utilizing the scientific literature.

Concerning the extension of the "reverse path" spectral method towards multi-d.o.f. systems, there are two distinct approaches. Each approach is formulated from the generalized set of coupled differential equations of motion; see equations (1a–c) and (2) of reference [2] and equations (1)–(3) of reference [5]. These equations are essentially Newton's second law of motion applied to a generic non-linear vibration system. Consequently, initial formulation of the methods should be similar. However, beyond these basic governing equations that set the stage for further analysis, there exist